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A Note on Left Regular Semiring

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ABSTRACT
In this paper we have focused on the additive and multiplicative identity 'e' and determine the additive and multiplicative semigroups. Here we established that, A semiring S in which (S, +) and (S, •) are left singular semigroups, then S is a left regular semiring. We have framed an example for this proposition by considering a two element set.
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I. INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces and the like. This provides sufficient motivation to researchers to review various concepts and results.

The study of rings which are special semirings shows that the multiplicative structures are quite independent though their additive structures are abelian groups. However, in semirings it is possible to derive the additive structures from their special multiplicative structures.

II. PRELIMINARIES

Definition 2.1:
A triple (S, +, •) is a semiring if S is a non-empty set and "+, •" are binary operations on S satisfying that
(i) The additive reduct (S, +) and the multiplicative reduct (S, •) are semigroups.
(ii) a (b + c) = ab + ac and (b + c) a = ba + ca, for all a, b, c in S.

Definition 2.2:
A semigroup (S, •) is known as quasi separative if $a^2 = ab = ba = b^2 \Rightarrow a = b$ for all a, b in S.

Definition 2.3:
A semigroup (S, +) is said to be weakly separative if $a + a = a + b = b + b \Rightarrow a = b$ for all a, b in S.

Definition 2.4:
A left regular semiring is a semiring in which a + ab + b = a for all a, b in S.

Definition 2.5:
Zeroid of a semiring S is the set of all **a** in S such that **a + b = b or b + a = b** for some **b** in S, we may also this use this term as the zeroid of (S, +).

Definition 2.6:
A viterbi semiring is a semiring in which S is additively idempotent and multiplicatively subidempotent. i.e., $a + a = a$ and $a + a^2 = a$ for all 'a' in S.

Definition 2.7:
An element a of a semiring S is completely regular if there exists an element x in S such that (A) a + x + a = a (B) a + x = x + a and (C) a (a + x) = a + x
Naturally, we call a semiring S is completely regular if every element a of S is completely regular.

Definition 2.8:
A C – semiring is a semiring in which
(i) (S, +) and (S, ·) are commutative monoids.
(ii) a (b + c) = ab + ac and (b + c) a = ba + ca for every a, b, c in S
(iii) a.0 = 0.a = 0
(iv) (S, +) is a band and 1 is the absorbing element of '+'.

Definition 2.9:
A semigroup (S, •) is left (right) singular if ab = a (ab = b) for all a, b in S.
A semigroup (S, +) is left (right) singular if a + b = a (a + b = b) for all a, b in S.

Definition 2.10:
In a totally ordered semiring (S, +, •, ≤), (S, +, ≤) is positively totally ordered (p.t.o), if a + b ≥ a, b for all a, b in S.

III. STRUCTURES OF LEFT REGULAR SEMIRING

Theorem 3.1: If S is a left regular semiring and S contains a multiplicative identity 'e' which is also an additive identity, then (S, ·) is quasi separative.

Proof: Let e be the multiplicative identity which is also additive identity
Since S is left regular semiring then a + ab + b = a for all a, b in S.
To prove that (S, ·) is quasi separative i.e., $a^2 = ab = ba = b^2 \Rightarrow a = b$
Let $a^2 = ab = a (e + b) = a + (a + e) b = a + ab + b = a$
Similarly $b^2 = ba = b (e + a) = b + (b + e) a = b + ba + a = b$
Thus $a^2 = a$ and $b^2 = b$
We proved that if $a^2 = ab = ba = b^2$ implies that a = ab = ba = b
Therefore a = b
Hence (S, ·) is quasi separative

Corollary 3.2: Suppose S is a left regular semiring and S contains a multiplicative identity 'e' which is also an additive identity. Then (S, +) is weakly separative.

Proof: Given that S is a left regular semiring also 'e' is multiplicative identity and also additive identity
To prove that (S, +) is weakly separative
i.e $a + a = a + b = b + b \Rightarrow a = b$ for all a, b in S
Let a + b = b + b
Already we know that a left regular semiring with 'e' multiplicative identity which is also an additive identity then (S, +) is a band and also rectangular band thus we obtain a + b = a and b + b = b $\Rightarrow$ a = b
Hence (S, +) is weakly separative

Theorem 3.3: Let S be a left regular semiring and (S, +) be zeroid. Then S is a viterbi semiring.

Proof: Assume that S is a left regular semiring then a + ab + b = a for a, b in S
Also it was given that (S, +) is zeroid we have b + a = b
Let us take a + b + ab = a which implies a + b + a (b + a) = a
It again leads to $a + b + ab + a^2 = a \Rightarrow a + a^2 = a$
Since (S, +) is zeroid we have a + a = a
i.e S is multiplicatively subidemotent and additively idempotent
Thus S is a viterbi semiring

Theorem 3.4: If S is a C - semiring which is also left regular semiring then (S, +) is a completely regular semigroup

Proof: By hypothesis (S, +) is an absorbing then a + 1 = 1which implies ab + b = b
It can also be written as $a + ab + b = a + b \Rightarrow a = a + b$
$\Rightarrow a + a = a + b + a \Rightarrow a = a + b + a$
Form C – semiring we have (S, +) is commutative
Hence (S, +) is an completely regular semigroup

Proposition 3.5: A semiring S in which (S, +) and (S, •) are left singular semigroups, then S is a left regular semiring.

Proof: By hypothesis, (S, +) is left singular semigroup then a + b = a for all a, b in S
Also a + a = a for all a in S
Suppose (S, ·) is left singular semigroup it implies ab = a
$\Rightarrow$ a + ab = a + a $\Rightarrow$ a + ab + b = a + b
$\Rightarrow$ a + ab + b = a

Example 3.6: The following example is evident for the above proposition.

+	**a**	**b**	**.**	**a**	**b**
a	a	a	**a**	a	a
b	a	b	**b**	b	b

Theorem 3.7: If S is a totally ordered left regular semiring and (S, +) is p.t.o then a + b = a.

Proof: Assume that (S, +) is positively totally ordered then a + b ≥ a and b
Suppose a + ab ≥ a it implies a + ab + b ≥ a + b $\Rightarrow$ a ≥ a + b ≥ a
Therefore a + b = a

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